THEORY FOR THREE PATH EXPERIMENTS

SUBMITTED
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Abstract

The theoretical framework for an atomic slit experiment, a variation of the triple slit experiment, is developed. Such experiments test the validity of the Born rule according to which, interference fringes in the triple-slit experiment can be written in terms of fringes observed in various double and single slit experiments. The goal is to use precision of atomic experiments to improve previous experimental bounds. However, general theories and experimental procedures applicable to multi-slit experiments with different physical systems are not directly translatable. We derive ideas on how to experimentally implement a tritter and blocker, the necessary components in an atomic slit experiment.

Contents

Chapter 1

Introduction

The Born rule is essential to quantum mechanics ever since it was conceived by Max Born in 1929 [1]. It states that for any quantum mechanical system, the probability for a measurement outcome is given by the corresponding probability amplitude squared. Although it is fundamentally at the core of quantum mechanics, it has yet to be thoroughly explored and fully verified. Building on the previous work of Sorkin [2], one direct way to probe the Born rule is to look at multi-path interference terms.

Several experiments have implemented the proposal of Sorkin. Most of them were variations of multi-path diffraction experiments [3] [4] [5] [6] and any system that involves interference can be used to test it. In the present thesis, we consider an atomic analog for these diffraction experiments, which we call the atomic slit experiment. Slits from diffraction experiments can be represented by different atomic states, and laser pulses can be used to carefully control them. The main motivation to such an approach is to extend the precision and accuracy of atomic interferometers [7] [8] to these experiments. This is similar to a previous experiment by Park et al. [9] that has hitherto the tightest experimental bounds.

In the following chapters, we present the theoretical analysis of three path experiments and their relevance to the Born rule. The goal is to develop a theoretical framework for the implementation of an atomic slit experiment.

Specifically, the formalism of the Born rule and its consequences are discussed in chapter 2. Certain important assumptions and approximations are also highlighted.

In chapter 3, we will broadly describe the general class of experiments that we refer to as three path experiments, with the triple slit experiment as a reference point. A method to quantify the violation of the Born rule and the proposal of an atomic slit experiment is explained.

Chapter 4 examines the theory of oscillations between atomic states and hence derives a schematic to obtain an equal superposition of three states, also referred to as a tritter.

Chapter 5 details the atomic analog for blocking slits in the triple slit experiment. Possible implementations are discussed, although a physical realisation of such an operation has not been performed.

Chapter 2

Born's Rule

2.1 Formalism

Formally, Born's rule states that for a quantum mechanical state specified by its wavefunction $\psi(\mathbf{r}, t)$, the probability p($\mathbf{r}$,t) that a particle lies in the volume element d^3r located at $\mathbf{r}$ and at time t is given by:

$$\begin{aligned} p(\mathbf{r}, t) &= \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t)d^3r \\ &= |\psi(\mathbf{r}, t)|^2 \, d^3r. \end{aligned} \tag{2.1}$$

Due to the quadratic nature, a subtle consequence is that third order and higher interference terms do not affect the probability function. To see this, we will use a short thought experiment.

Consider three distinct sources labelled A, B and C, emitting particles with some measurable quantity. The system should also be on the scale such that quantum mechanical effects are relevant. Source A, B and C emits particles with wavefunctions ψ_A, ψ_B, and ψ_C respectively. Thus, the whole system is described by the wavefunction:

$$\psi_{ABC} = \psi_A + \psi_B + \psi_C. \tag{2.2}$$

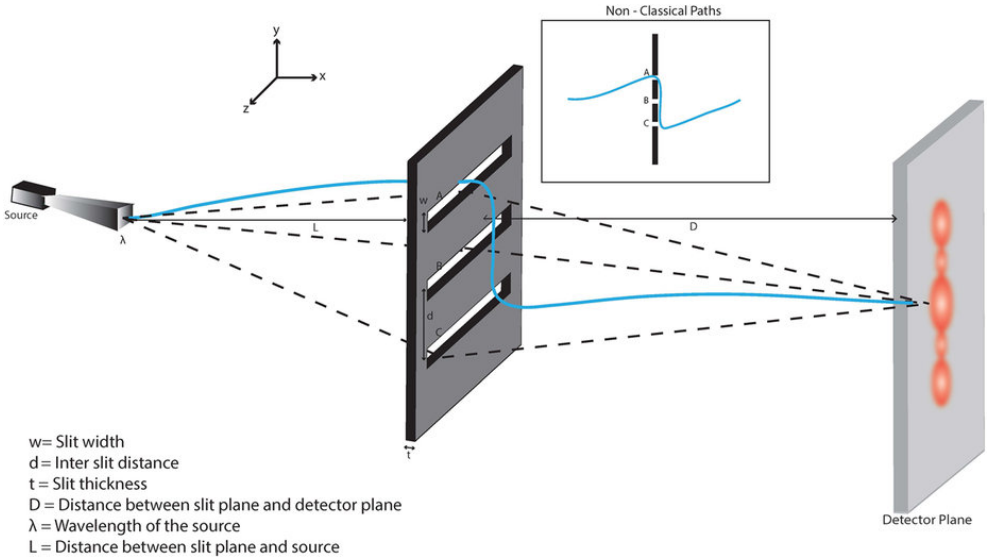

Figure 2.1: A triple slit experiment with a non-classical path in the solid line while the classical paths are in the dotted lines [11]. Under the Fraunhofer regime, the contributions of the non-classical paths are negligible.

It should be noted for correctness, that the above sum is not exactly true [10] [11]. Using the Feynman path integral formalism [12], the exact sum should also include a ψ_{NC} term, which is the contribution from non-classical paths, which are paths that do not extremise classical action. These could be looped or unusual paths like in Fig. 2.1. However, under reasonable conditions, like the detector being in the far field and very thin slits, these non-classical contributions become negligible [13].

By the Born rule, the probability that we would measure a particle from any one of these sources would be given by:

$$\begin{aligned} p(ABC) &= |\psi_A + \psi_B + \psi_C|^2 \\ &= (\psi_A + \psi_B + \psi_C)(\psi_A^* + \psi_B^* + \psi_C^*) \\ &= |\psi_A|^2 + |\psi_B|^2 + |\psi_C|^2 + \psi_A^*\psi_B + \psi_B^*\psi_C + \psi_C^*\psi_A + c.c. \end{aligned} \tag{2.3}$$

Only pairwise combinations of $\psi_i^* \psi_j$, for $i, j = A, B, C$, are present. The second order interference terms are the ones with $i \neq j$. Clearly, higher order interference terms are absent. Although this is shown only for three sources, it can easily be seen that increasing the number of sources will not introduce higher order terms. With this observation in mind, we can use this three source framework to investigate the Born rule.

Chapter 3

Three Path Experiments

A three path experiment is one where a particle propagates via three paths before being recombined and measured, but inferring the exact path taken is impossible. The output signal from these different paths can interfere constructively or destructively with each other, but by Born's rule, any three-way or higher interference should be absent.

3.1 Experiment Schematic

An experimental scheme for a three path experiment is depicted in Fig. 3.1.

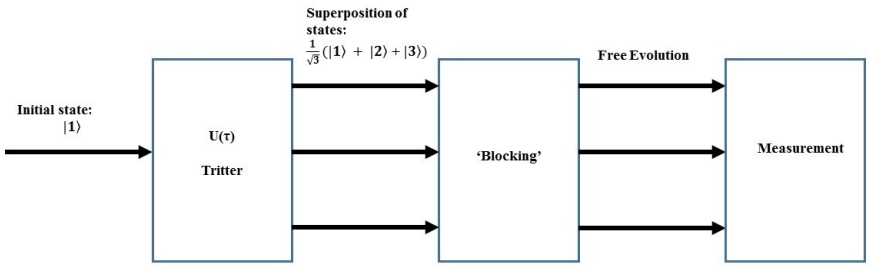

Figure 3.1: Experimental Schematic of a general three path experiment

There are three main components to this scheme, mainly a tritter, a blocker and

some measurement at the end. We will first consider them as abstract processes in the most general sense without any particular physical realisations. The purpose of each component is discussed briefly before describing them and their possible realisations in detail in the next chapters.

The tritter functions essentially to create three indistinguishable paths. In principal, this can be realised in numerous ways, like a three-way beam splitter or even simply three seperate laser sources. We will mainly focus on states in some driving field and obtain a rather interesting tritter.

The blocker allows us to "block" certain paths in the system, essentially removing those states. One may either redistribute the blocked states to the remaining states or completely eliminate them entirely from the system. Although an actual realisation of either type of blockers are not yet completed in detail, a few ideas are proposed and discussed, together with their limitations and drawbacks in Chapter 4.

Any and every measurement must follow the Born rule if it is true. As such, in the theoretical discussions, we will refer to making a measurement as corresponding to some abstract and general operator $|m\rangle \langle m|$.

3.2 Triple Slit Experiment

One such experiment would be the triple slit experiment, consisting of a coherent light source, three slits on an optically opaque plate separated spatially by some finite distance and a detector arranged as in Fig. 3.2. The slits can be treated as independent and coherent sources as the light passes through them, essentially functioning as the tritter. Some form of shutter or secondary plate could be used as the blocker [3] [4], while the measurement is some highly sensitive photon detector.

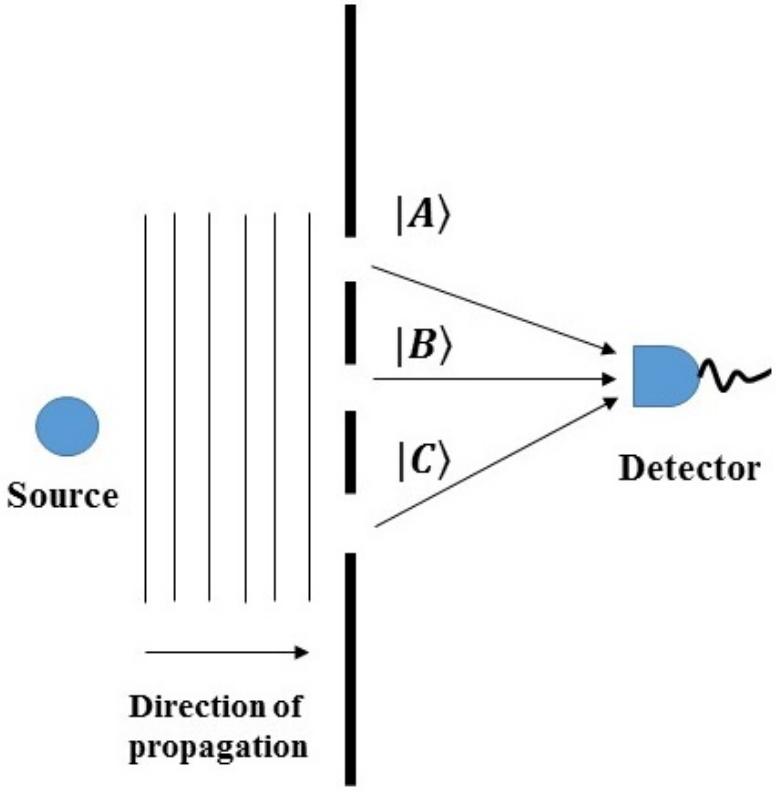

Figure 3.2: The triple slit experiment. Depending on the position of the detector relative to the slits, the signal measured depends on the interference between the slits. In principle, the interference can either be constructive, destructive or somewhere in between, depending on the path differences.

3.3 Sorkin's Quantity

To verify the Born rule, we require some method to quantify interference. We use a quantity defined by Sorkin in previous works [2].

The first order Sorkin term for a slit labelled A is defined as:

$$S_1(A) := p_A, \tag{3.1}$$

where p_A refers to the probability of detection of a particle having travelled only through slit(s) denoted by the subscript A, or that we have blocked all other slits except A. For the remainder of the discussion, subscripts of A,B,C will refer to the labelled slits accordingly, unless otherwise stated. S_1 is not zero for non-trivial measurements.

For the second order Sorkin quantity for slits A and B,

$$S_2(A, B) := p_{AB} - p_A - p_B. \tag{3.2}$$

This term non-zero and is a consequence of the additivity rules for probabilities in Quantum Mechanics. If we write this down in terms of the wave functions, we get

$$\begin{aligned} p_{AB} - p_A - p_B &= |\psi_A + \psi_B|^2 - |\psi_A|^2 - |\psi_B|^2 \\ &= \psi_A \psi_B^* + \psi_A^* \psi_B, \end{aligned} \tag{3.3}$$

which is exactly the second order interference term.

However, the third order term for slits A, B and C,

$$S_3(A, B, C) := p_{ABC} - p_{AB} - p_{BC} - p_{AC} + p_A + p_B + p_C, \tag{3.4}$$

is necessarily 0 by the Born rule. We can show this by directly comparing equations (2.3) and (3.3) to observe that,

$$\begin{aligned} p_{ABC} &= p_A + p_B + p_C + S_2(A, B) + S_2(B, C) + S_2(A, C) \\ &= p_{AB} + p_{BC} + p_{AC} - p_A - p_B - p_C, \end{aligned} \tag{3.5}$$

which clearly makes the third order Sorkin quantity vanish.

Thus, we now have a rubric to verify the Born rule experimentally. By measuring these probabilities of detection for different combinations of blocked slits, we can calculate the Sorkin quantity and observe how close to zero it is.

3.4 Atomic Slits

Deviating from the triple slit experiment, a modified experiment using atomic energy levels could be performed. With the Born's rule being essential to all of quantum mechanics, it is important to check its validity in different setups.

As a comparison, three energy levels can be thought of each as a slit. To distinguish between the triple slit, subscripts 1,2 and 3 are used instead of A, B and C when referring to the atomic slits. Instead of interference in the spatial domain, an atomic analog would generate similar interference patterns in the time domain.

Blocking a slit is considerably more complicated to replicate in an atomic slit experiment. To 'block off' an energy level, the most straightforward way would be to make a measurement on the system to check if the energy level is occupied and only keep results that are not. This is similar to the triple slit where the blockage can be viewed as a measurement. To do so, one may shine a resonant laser that optically pumps the target level to some excited state which then decays back to the any of the three energy levels through spontaneous emission. After a sufficiently long duration, the population at the target level will be completely transferred to the other unblocked levels and be effectively 'blocked off'.

The process of 'blocking' has certain implications that need to be addressed. In the physical blocking of a slit, the contributions of that particular slit is completely removed from the screen while the optical blocking transfers it to the other unblocked levels. This complicates the blocking process because the repopulation to the other levels will be incoherent and the transferring would finally cause the Sorkin quantity to be non-zero. The effects of the incoherence will be further discussed in the next section.

Chapter 4

Tritter

The idea of a three state splitter, or tritter (Fig. 4.1) , is to begin with one of the three states and output an equal and coherent superposition of the three states. We focus on atomic energy levels as our states. To understand how to realise an atomic tritter, we will build on the theoretical foundations on atomic state transitions.

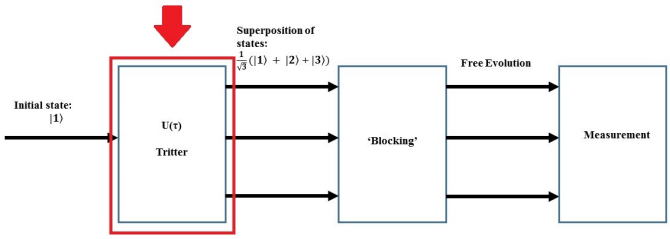

Figure 4.1: The tritter component of the three path experiment. It takes an initial state $|1\rangle$ and outputs a superposition of all three states.

4.1 Rabi Oscillations

We begin with the simplest case of a two level system driven by near resonant radiation. A semi-classical approach is used to describe the oscillation of the state

populations, and the result is the well-known Rabi oscillations. The method used is similar to that found in Allen, L. and Eberly, J.H.'s "Optical Resonance and Two-level Atoms" [14].

We begin by defining relevant parameters to the derivation. The two levels, $|1\rangle$ and $|2\rangle$, form an orthonormal basis for the system, so $\langle i|j\rangle = \delta_{ij}$ for $i, j = 1, 2$; with a resonant frequency of ω_0 between them. A driving field is detuned by a small amount, Δ, from the resonant frequency, meaning that it has a frequency of $\omega = \omega_0 + \Delta$. These parameters are illustrated in Fig. 4.2.

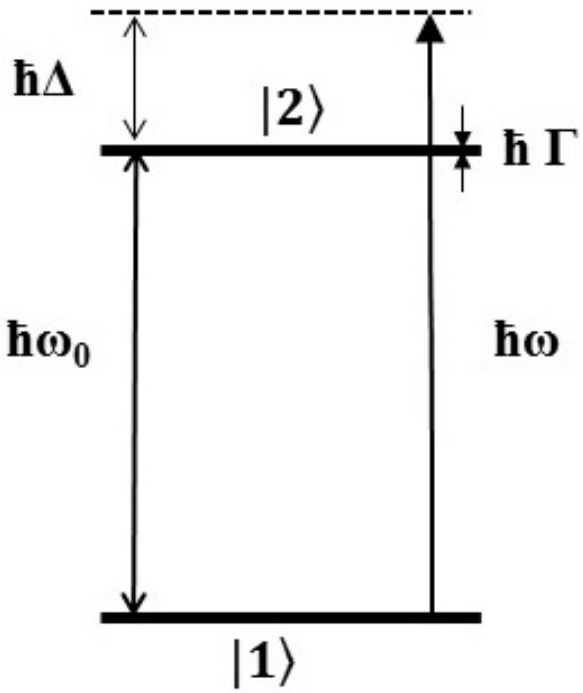

Figure 4.2: Two-level system with a driving laser

In general, our two-level quantum state can be expressed as:

$$|\psi(t)\rangle = c_1(t)\,|1\rangle + c_2(t)\,|2\rangle\,, \tag{4.1}$$

$c_1(t)$ and $c_2(t)$ satisfy the normalization condition:

$$|c_1(t)|^2 + |c_2(t)|^2 = 1. \tag{4.2}$$

It can also be expressed as a two-vector with components $\psi_i = \langle i|\psi\rangle$:

$$\psi = \begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \end{pmatrix} = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}. \tag{4.3}$$

4.1.1 Hamiltonian of the two-level system

To understand how the two-level system evolves in time, we determine its Hamiltonian, $\hat{H}$, which is the operator corresponding to the total energy of the system, and solve the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(t) = \hat{H}\psi(t). \tag{4.4}$$

We can seperate the Hamiltonian of our system into two components, the bare Hamiltonian, which is the Hamiltonian in the absence of the driving field, $\hat{H}_0$, and the interaction Hamiltonian, which only considers the effect of the driving field, $\hat{H}_{int}$. Or simply,

$$\hat{H} = \hat{H}_0 + \hat{H}_{int}. \tag{4.5}$$

Without the driving field, the states $|1\rangle$ and $|2\rangle$ are the eigenstates of the bare

Hamiltonian, H_0. Thus, H_0 satisfies the following equations:

$$\hat{H}_0 \left|1\right\rangle = 0 \left|1\right\rangle, \tag{4.6a}$$

$$\hat{H}_0 \left|2\right\rangle = \hbar\omega_0 \left|2\right\rangle, \tag{4.6b}$$

where we choose the energy of the ground state $|1\rangle$ to be zero.
The bare Hamiltonian has a matrix representation, with respect to the energy eigenstates $|1\rangle$ and $|2\rangle$, with components $H^0_{i,j} = \langle i|\hat{H}_0|j\rangle$:

$$\begin{aligned} H_0 &= \begin{pmatrix} \langle 1|\hat{H}_0|1\rangle & \langle 1|\hat{H}_0|2\rangle \\ \langle 2|\hat{H}_0|1\rangle & \langle 2|\hat{H}_0|2\rangle \end{pmatrix} \\ &= \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega_0 \end{pmatrix}. \end{aligned} \tag{4.7}$$

The effect of the electromagnetic driving field is to couple the two states $|1\rangle$ and $|2\rangle$. A transition dipole moment is induced and the driving field interacts with it. This can be represented as another 2 x 2 matrix given by:

$$\begin{aligned} H_{int} &= \hbar \begin{pmatrix} 0 & \Omega cos(\omega t) \\ \Omega^* cos(\omega t) & 0 \end{pmatrix} \\ &= \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega(e^{i\omega t} + e^{-i\omega t}) \\ \Omega^*(e^{i\omega t} + e^{-i\omega t}) & 0 \end{pmatrix}. \end{aligned} \tag{4.8a}$$

where the Rabi frequency, Ω, is related to the induced dipole moment μ and amplitude of the electromagnetic driving field E_0 by the following equation:

$$\hbar\Omega = \mu E_0. \tag{4.9}$$

Now that we have defined both the bare and interaction Hamiltonian, we can construct our total Hamiltonian, $H = H_0 + H_{int}$ (using equations (4.7) and (4.8)

):

$$H = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega(e^{i\omega t} + e^{-i\omega t}) \\ \Omega^*(e^{i\omega t} + e^{-i\omega t}) & 2\omega_0 \end{pmatrix}. \tag{4.10}$$

The total Hamiltonian in this form is not immediately useful to us because it contains a time dependence. To further simplify it, we make a transformation from the Schrödinger picture to the interaction picture.

4.1.2 Interaction Picture

For any Hamiltonian in the Schrödinger picture, we may divide it into two parts such that:

$$H_S = H_{0,S} + H_{1,S}. \tag{4.11}$$

Then, any operator, $A_I(t)$, in the interaction picture, including the Hamiltonian, with a corresponding operator in the Schrödinger picture, $A_S(t)$, is defined as:

$$A_I(t) = e^{iH_{0,S}t/\hbar} A_S(t) e^{-iH_{0,S}t/\hbar}. \tag{4.12}$$

In particular, the perturbation Hamiltonian is given by:

$$H_{1,I}(t) = e^{iH_{0,S}t/\hbar} H_{1,S}(t) e^{-iH_{0,S}t/\hbar}. \tag{4.13}$$

In the interaction picture, the Schrödinger equation is transformed into the Schwinger-Tomonaga equation:

$$i\hbar\frac{\partial}{\partial t}\psi_I(t) = H_{1,I}(t)\psi_I(t). \tag{4.14}$$

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4.1.3 Solving in the interaction picture

Returning to our specific two-level system, we define $H_{0,S}$ as:

$$H_{0,S} = \hbar \begin{pmatrix} 0 & 0 \\ 0 & \omega \end{pmatrix}, \tag{4.15}$$

which, using equations (4.11) and (4.13), gives us the perturbation Hamiltonian,

$$\begin{aligned} H_{1,I} &= e^{iH_{0,S}t/\hbar}(H - H_{0,S}(t))e^{-iH_{0,S}t/\hbar} \\ &= \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega t} \end{pmatrix} \begin{pmatrix} 0 & \Omega(e^{i\omega t} + e^{-i\omega t}) \\ \Omega^*(e^{i\omega t} + e^{-i\omega t}) & 2(\omega_0 - \omega) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\omega t} \end{pmatrix} && \text{(4.16a)} \\ &= \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega(1 + e^{-i2\omega t}) \\ \Omega^*(1 + e^{-i2\omega t}) & -2\Delta \end{pmatrix}. && \text{(4.16b)} \end{aligned}$$

It is here that we make the Rotating Wave Approximation (RWA) [15] which drops the terms that are twice the driving frequency. The approximation is justified because our driving field is near resonance, so rapidly oscillating terms average to zero on an appreciable time scale. This simplifies our perturbation Hamiltonian to:

$$H_{1,I} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega \\ \Omega^* & -2\Delta \end{pmatrix}. \tag{4.17}$$

Using this Hamiltonian in the Schwinger-Tomonaga equation (4.14), yields the following system of equations:

$$i\hbar \begin{pmatrix} \dot{c_1}(t) \\ \dot{c_2}(t) \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega \\ \Omega^* & -2\Delta \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}. \tag{4.18a}$$

The Rabi frequency, Ω, can be set to be real by adjusting the phase of the driving field to be zero. Using the initial conditions that the system starts with all the population in state 1, i.e. $c_1(0) = 1$ and $c_2(0) = 0$, the solution to the set of

differential equations in (4.18) is:

$$\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = e^{\frac{it\Delta}{2}} \begin{pmatrix} \cos(\frac{\Omega_R t}{2}) - i\frac{\Delta}{\Omega_R}\sin(\frac{\Omega_R t}{2}) \\ -i\frac{\Omega}{\Omega_R}\sin(\frac{\Omega_R t}{2}) \end{pmatrix}, \tag{4.19}$$

where $\Omega_R^2 = \Omega^2 + \Delta^2$ is the total Rabi frequency.

It is worth noting that these solutions are in the interaction picture. However, we can show that the probabilities to be in the states $|1\rangle$ and $|2\rangle$ are invariant under this transformation. In the interaction picture,

$$|\psi_I\rangle = e^{iH_{0,S}t/\hbar}\,|\psi_S\rangle\,, \tag{4.20}$$

so the transformation of states between the Schrödinger and interaction picture is just applying a global phase.

4.2 Coherent Raman Transitions

Suppose now that instead of just a two-level system, we have more energy levels. It turns out that we can configure such systems to produce unique transitions and couplings between the states. We begin by analysing a system with just one more additional energy level, the three-level atom with driving fields made up of two lasers in a Λ configuration as shown in Fig. 4.3. The following discussion is based on the work of Dotsenko [16]. Using the same convention as the previous section, any quantum state of this system can be expressed as:

$$|\psi(t)\rangle = c_1(t)\,|1\rangle + c_2(t)\,|2\rangle + c_e(t)\,|e\rangle\,. \tag{4.21}$$

In vector form, this is also

$$\psi = \begin{pmatrix} c_1(t) \\ c_2(t) \\ c_e(t) \end{pmatrix}. \tag{4.22}$$

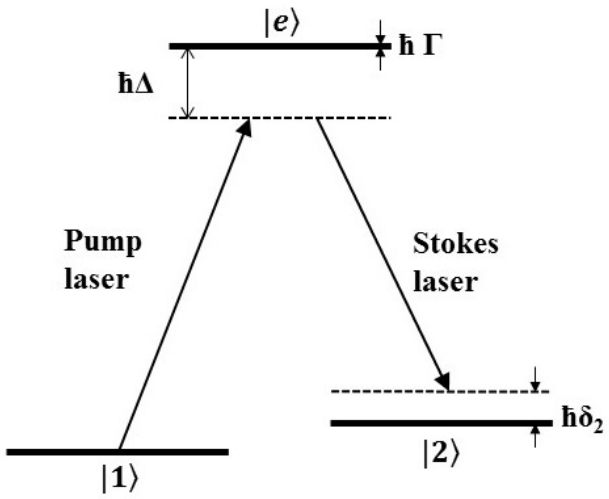

Figure 4.3: Three-level system in the Λ configuration

The energy levels $|1\rangle$ and $|e\rangle$ are coupled by the pump laser, while $|2\rangle$ and $|e\rangle$ are coupled by the Stokes laser. The net effect is that $|1\rangle$ and $|2\rangle$ are coherently coupled by both lasers. This is the coherent Raman transition. For this to happen, the inequaility $\Delta \gg \Gamma$ has to be satisfied, where Γ is the linewidth of the excited level and is related to the rate of spontaneous emission. Since spontaneous emission is an incoherent process, we must have the line width to be negligible relative to all other parameters.

An interesting property of coherent Raman transitions is that with a sufficiently large detuning, Δ, of the Raman beams from the excited state, the excited state does not get populated and a virtual state is instead involved in the transitions.

4.2.1 Hamiltonian of the Λ sytem

Like the Rabi problem, we begin by determing the Hamiltonian of the system. We can view the Raman transition as two seperate Rabi oscillations, one between the levels $|1\rangle$ and $|e\rangle$, characterised by the Rabi frequency Ω_1, and the other between $|2\rangle$ and $|e\rangle$, characterised by Ω_2. Using the results from the previous section, we then have the matrix representation for the Hamiltonian given by (note that our detunings now have different sign):

$$\hat{H} = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & \Omega_1 \\ 0 & 2\delta & \Omega_2 \\ \Omega_1^* & \Omega_2^* & 2\Delta \end{pmatrix}. \tag{4.23a}$$

4.2.2 Solutions to the Λ system

Solving the Schrödinger equation is then equivalent to solving the following system of equations:

$$i\hbar\begin{pmatrix} \dot{c_1}(t) \\ \dot{c_2}(t) \\ \dot{c_e}(t) \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & \Omega_1 \\ 0 & 2\delta & \Omega_2 \\ \Omega_1^* & \Omega_2^* & 2\Delta \end{pmatrix}\begin{pmatrix} c_1(t) \\ c_2(t) \\ c_e(t) \end{pmatrix}. \tag{4.24}$$

Or, more explicitly,

$$\begin{cases} i\dot{c_1}(t) &= \frac{1}{2}\Omega_1 c_e(t), \\ i\dot{c_2}(t) &= \frac{1}{2}\Omega_2 c_e(t) + \delta c_2(t), \\ i\dot{c_e}(t) &= \frac{1}{2}\Omega_1^* c_1(t) + \frac{1}{2}\Omega_2^* c_2(t) + \Delta c_e(t). \end{cases} \tag{4.25}$$

While this system is solvable, the solutions are significantly more complicated than the Rabi oscillations. As such, one would look for certain limiting conditions and approximations to simplify the system.

As before, we can set our Rabi frequencies to be real by adjusting the relative

phases of the driving fields to be zero. Also, note that $\dot{c_e}(t)$ is almost twice of $\dot{c_1}(t)$ and $\dot{c_2}(t)$. Similar to the RWA, we would expect that $\dot{c_e}(t)$ averages to zero over many oscillations. This effectively reduces our system to a two-level one and is also known as adiabatic elimination. Our reduced system is now:

$$\begin{cases} i\dot{c_1}(t) &= -\frac{1}{4\Delta}\Omega_1(\Omega_1 c_1(t) + \Omega_2 c_2(t)), \\ i\dot{c_2}(t) &= -\frac{1}{4\Delta}\Omega_2(\Omega_1 c_1(t) + \Omega_2 c_2(t)) + \delta c_2(t). \end{cases} \tag{4.26}$$

With a new effective Hamiltonian written as:

$$H_{eff} = -\frac{\hbar}{4\Delta}\begin{pmatrix} \Omega_1^2 & \Omega_1\Omega_2 \\ \Omega_1\Omega_2 & \Omega_2^2 - 4\delta\Delta \end{pmatrix}. \tag{4.27}$$

Interestingly, our two states $|1\rangle$ and $|2\rangle$ are now coupled through the two-step process of absorption and emission, as observed from the off-diagonal elements in this new Hamiltonian. Solving (4.26) using the initial conditions that $c_1(0) = 1$ and $c_2(0) = 0$ gives us the time dependence of the system:

$$|c_1(t)|^2 = 1 - \Lambda \sin^2(\frac{\Omega_0}{2}t), \tag{4.28a}$$

$$|c_2(t)|^2 = \Lambda \sin^2(\frac{\Omega_0}{2}t), \tag{4.28b}$$

where $\Omega_0 = \frac{\Omega_1\Omega_2}{2\Delta}$ is the resonance Rabi frequency, $\Omega_R = \sqrt{\Omega_0^2 + \delta^2}$ is the generalised Rabi frequency and $\Lambda = \frac{\Omega_0^2}{\Omega_R^2}$ is the amplitude of the population oscillation.

4.3 Tripod System

From the description of Raman transitions, we understand how to couple two stable ground states by using a three level system. A natural extension would then be to couple three states by using a four level system. This is the main idea of the tritter. The four level system, or tripod system is shown in Fig. 4.4.

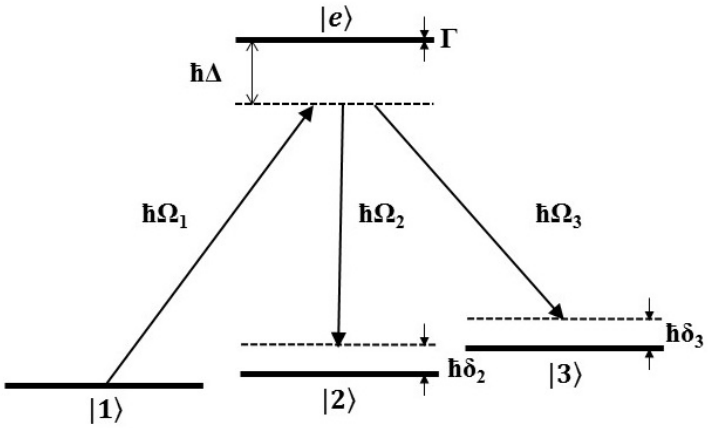

Figure 4.4: Four-level system, or Tripod system.

As before, any quantum state in this system can be written as:

$$|\psi(t)\rangle = c_1(t)\,|1\rangle + c_2(t)\,|2\rangle + c_3(t)\,|3\rangle + c_e(t)\,|e\rangle\,, \tag{4.29}$$

or as a vector,

$$\psi(t) = \begin{pmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \\ c_e(t) \end{pmatrix}. \tag{4.30}$$

For our tritter to work, we must couple the three ground states together without populating the excited state. Thus, the inequality $\Delta \gg \Gamma$ must hold.

When comparing with the triple slit experiment, each slit is essentially identical to another. To keep this similarity, the three ground states should be degenerate energy levels, or close enough, in the scale of the energy gap between the excited state, to be considered degenerate. One potential problem would be that for degenerate energy levels, the same laser could have the same effect on all three states because it corresponds to the same energy gap between the virtual state. However, we can get past this by considering angular momentum selection rules.

For $|e\rangle$ with some fixed orbital angular momentum number, we can use polarised lasers in such a way that only excitation via laser 1 is allowed for $|1\rangle$, etc. As such, the maximum number of equal energy states that can be coupled is three, corresponding to the total number of polarisation states of light, namely linear, circular positive and circular negative.

4.3.1 Hamiltonian for the Tripod system

The total Hamiltonian following derivations from the previous section is given by:

$$\hat{H} = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & 0 & \Omega_1 \\ 0 & 2\delta_2 & 0 & \Omega_2 \\ 0 & 0 & 2\delta_2 & \Omega_3 \\ \Omega_1 & \Omega_2 & \Omega_3 & 2\Delta \end{pmatrix}, \tag{4.31}$$

And the corresponding Schrödinger equation to solve is:

$$i\hbar\begin{pmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \\ \dot{c}_3(t) \\ \dot{c}_e(t) \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & 0 & \Omega_1 \\ 0 & 2\delta_2 & 0 & \Omega_2 \\ 0 & 0 & 2\delta_2 & \Omega_3 \\ \Omega_1 & \Omega_2 & \Omega_3 & 2\Delta \end{pmatrix}\begin{pmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \\ c_e(t) \end{pmatrix}. \tag{4.32}$$

4.3.2 Solutions to the Tripod system

This is essentially a system of 4 coupled differential equations with 8 variables, which is significantly more complicated than both the Raman transitions and Rabi oscillations. As such, we will look for more limiting conditions and approximations to simplify the system to a more manageable form.

Firstly, we can use the adiabatic elimination like in the Raman transitions to reduce our system to 3 coupled differential equations. We can tune the lasers

such that:

$$\delta_2 = \delta_3 = 0. \tag{4.33}$$

This also gives the strongest coupling (see equation (**??**)).

Recalling the relation between the Rabi frequency to the amplitude of the driving field in (4.9), we can also choose the laser power such that:

$$\Omega_1 = \Omega_2 = \Omega_3 = \Omega. \tag{4.34}$$

Thus, our system of equations now takes the simplified form:

$$\begin{cases} i\dot{c}_1(t) &= -\frac{1}{4\Delta}\Omega^2(c_1(t) + c_2(t) + c_3(t)), \\ i\dot{c}_2(t) &= -\frac{1}{4\Delta}\Omega^2(c_1(t) + c_2(t) + c_3(t)), \\ i\dot{c}_3(t) &= -\frac{1}{4\Delta}\Omega^2(c_1(t) + c_2(t) + c_3(t)). \end{cases} \tag{4.35}$$

With the effective Hamiltonian being:

$$H_{eff} = -\frac{1}{4\Delta}\begin{pmatrix} \Omega^2 & \Omega^2 & \Omega^2 \\ \Omega^2 & \Omega^2 & \Omega^2 \\ \Omega^2 & \Omega^2 & \Omega^2 \end{pmatrix}. \tag{4.36}$$

For consistency in the experiment, we will always start with a fixed initial state. Here, we always begin with a state population that is all in $|1\rangle$. This corresponds to using the initial state condition of $c_1(0) = 1$, $c_2(0) = 0$ and $c_3(0) = 0$, and so we compute the solution of (4.35):

$$c_1(t) = \frac{1}{3}[2 + \cos(\frac{3\Omega^2 t}{4\Delta}) - i\sin(\frac{3\Omega^2 t}{4\Delta})], \tag{4.37a}$$

$$c_2(t) = \frac{1}{3}[-1 + \cos(\frac{3\Omega^2 t}{4\Delta}) - i\sin(\frac{3\Omega^2 t}{4\Delta})], \tag{4.37b}$$

$$c_3(t) = \frac{1}{3}[-1 + \cos(\frac{3\Omega^2 t}{4\Delta}) - i\sin(\frac{3\Omega^2 t}{4\Delta})]. \tag{4.37c}$$

For symmetry reasons, $c_2(t) = c_3(t)$.

4.3.3 Realising the Tritter

The state populations oscillate in time according to these time dependence functions. We can check that the population of the system is conserved.

$$\begin{aligned} |c_1(t)|^2 + |c_2(t)|^2 + |c_3(t)|^2 &= \frac{1}{9}[4 + 4\cos(\frac{3\Omega^2 t}{4\Delta}) + 1 - 2\cos(\frac{3\Omega^2 t}{4\Delta}) + 1 + 1 - 2\cos(\frac{3\Omega^2 t}{4\Delta}) + 1] \\ &= \frac{1}{9}(9) = 1. \end{aligned} \tag{4.38}$$

For a fixed time τ, the tritter will output a fixed superposition of states. Then, if we want our tritter to evenly distribute to all 3 states, we require the condition that:

$$|c_1(\tau)|^2 = |c_2(\tau)|^2 = |c_3(\tau)|^2 = \frac{1}{3}. \tag{4.39}$$

We do not need to explicitly solve for $c_2(\tau)$ and $c_3(\tau)$ because they are equal and solutions will follow as long as we solve $c_1(\tau)$. To obtain the duration of τ, we solve the equation:

$$\begin{aligned} |c_1(\tau)|^2 &= \frac{1}{9}[4 + 4\cos(\frac{3\Omega^2 \tau}{4\Delta}) + \cos^2(\frac{3\Omega^2 \tau}{4\Delta}) + \sin^2(\frac{3\Omega^2 \tau}{4\Delta})] \\ &= \frac{1}{9}[5 + 4\cos(\frac{3\Omega^2 \tau}{4\Delta})] = \frac{1}{3}. \end{aligned} \tag{4.40}$$

The solution is:

$$\tau = \frac{8\pi\Delta}{9\Omega^2} + 2k\pi, \tag{4.41}$$

for some integer k. We choose $k = 0$ to have τ as short as possible.

Thus, we have realised the tritter by using this tripod system and a τ-pulse.

4.3.4 Tritter as a Unitary Operator

The tritter acts as some operator on the states $|1\rangle$, $|2\rangle$, and $|3\rangle$. Exactly what this operator is will be discussed in this section.

After solving for τ, we can now explicitly write the coefficients of the states after passing through the tritter:

$$c_1(\tau) = \frac{1}{\sqrt{3}} e^{i\frac{\pi}{6}}, \tag{4.42a}$$

$$c_2(\tau) = \frac{1}{\sqrt{3}} e^{i\frac{-7\pi}{6}}, \tag{4.42b}$$

$$c_3(\tau) = \frac{1}{\sqrt{3}} e^{i\frac{-7\pi}{6}}. \tag{4.42c}$$

Then, we can express the tritter operator as $\hat{T}$, where

$$\hat{T}\,|1\rangle = \frac{1}{\sqrt{3}} e^{i\frac{\pi}{6}}\,|1\rangle + \frac{1}{\sqrt{3}} e^{i\frac{-7\pi}{6}}\,|2\rangle + \frac{1}{\sqrt{3}} e^{i\frac{-7\pi}{6}}\,|3\rangle\,. \tag{4.43}$$

In fact, we can generalise $\hat{T}$ for any input state. It simply takes the input state, redistributes the population equally to all 3 states while introducing the phases $e^{i\tau_1}$ on the input and $e^{i\tau_2}$ on the other states, where

$$\tau_1 = \frac{\pi}{6}, \tag{4.44a}$$

$$\tau_2 = \frac{-7\pi}{6}. \tag{4.44b}$$

In other words,

$$\hat{T}\,|k\rangle = \frac{1}{\sqrt{3}} e^{i\tau_1}\,|k\rangle + \sum_{j\neq k} \frac{1}{\sqrt{3}} e^{i\tau_2}\,|j\rangle\,, \tag{4.45}$$

for $j, k = 1, 2, 3$.

Interestingly, $\hat{T}$ is also a unitary operator. To show this, we simply need to

show that $\hat{T}\hat{T}^\dagger = \mathbb{1}$. We begin by constructing $\hat{T}$.

$$\begin{aligned} \hat{T}_i j &= \langle i | \hat{T} | j \rangle , \\ \hat{T} &= \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\tau_1} & e^{i\tau_2} & e^{i\tau_2} \\ e^{i\tau_2} & e^{i\tau_1} & e^{i\tau_2} \\ e^{i\tau_2} & e^{i\tau_2} & e^{i\tau_1}. \end{pmatrix} \end{aligned} \tag{4.46a}$$

Taking note that,

$$e^{i\tau_1} \cdot e^{-i\tau_2} = e^{i\frac{8\pi}{6}}, \tag{4.47}$$

and,

$$e^{i\frac{8\pi}{6}} + e^{-i\frac{8\pi}{6}} = -1. \tag{4.48}$$

It is easy to see that,

$$\begin{aligned} \hat{T}\hat{T}^\dagger &= \frac{1}{3} \begin{pmatrix} e^{i\tau_1} & e^{i\tau_2} & e^{i\tau_2} \\ e^{i\tau_2} & e^{i\tau_1} & e^{i\tau_2} \\ e^{i\tau_2} & e^{i\tau_2} & e^{i\tau_1} \end{pmatrix} \begin{pmatrix} e^{i\tau_1} & e^{i\tau_2} & e^{i\tau_2} \\ e^{i\tau_2} & e^{i\tau_1} & e^{i\tau_2} \\ e^{i\tau_2} & e^{i\tau_2} & e^{i\tau_1} \end{pmatrix}^\dagger \\ &= \frac{1}{3} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \\ &= \mathbb{1}, \end{aligned} \tag{4.49}$$

as expected for a unitary operation.

Chapter 5

Blocking of States

The next component in the experimental schematic is the realisation of the blocker (Fig. 5.1 in our atomic slit experiment. In the most general sense, such a process would take in any input of the superposition of states $|1\rangle$, $|2\rangle$ and $|3\rangle$, and remove particular states from the ouput.

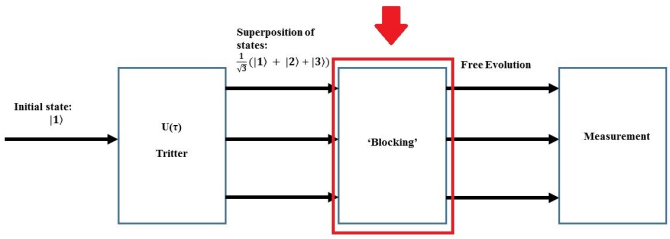

Figure 5.1: The blocker component of the three path experiment. It takes in an input of a superposition of states and selectively "blocks" certain states.

In other words, we can represent it as an operator $\hat{B}_{i,j,k}$, where the subscript here indicates the states that are blocked and 0 means no state is blocked. For consistency, the indexes will be arranged in ascending order. Then, for example, blocking state $|1\rangle$ is equivalent to the operator $\hat{B}_{0,0,1}$ while blocking states $|1\rangle$ and $|2\rangle$ refers to the operator $\hat{B}_{0,1,2}$. The action of $\hat{B}_{0,0,1}$ on a superposition of $|1\rangle$, $|2\rangle$ and $|3\rangle$ is given below:

$$\hat{B}_{0,0,1}(c_1 |1\rangle + c_2 |2\rangle + c_3 |3\rangle) = c_2 |2\rangle + c_3 |3\rangle . \tag{5.1}$$

This is similar to the blocking of slits in the triple slit experiment, where the blocked photons do not reach the detector. We will refer to such a process as the elimination of states.

5.1 Elimination of States

In the triple slit case, the states are allowed to interfere in the spatial domain between the screen and the slits while the states in from the atomic slit using the elimination scheme only undergo free evolution. For the degenerate energy levels in our tripod system, this does not generate a phase difference as every state gains the same phase of $e^{-iEt_0/\hbar}$ where E is the energy of the state and t_0 is the duration of the evolution. No interaction zone is immediately apparent.

Consider now, the addition of another tritter between the blocking and the measurement. The second tritter here functions as an interaction zone that mixes the states. We will show that under this new schematic, the Sorkin quantity still vanishes.

From the previous chapter, we are clear how to states evolve after interaction with the first tritter. The operator here for the second tritter is essentially identical since we are using the same configuration. The main difference is now that the input state is no longer always the pure state $|1\rangle$. For the blocking of state $|1\rangle$ under the elimination scheme, the input state is then:

$$\begin{aligned} \psi_{23} &= \hat{B}_{0,0,1}(\frac{1}{\sqrt{3}}[e^{i\tau_1}|1\rangle + e^{i\tau_2}|2\rangle + e^{i\tau_2}|3\rangle] \\ &= \frac{1}{\sqrt{3}}[e^{i\tau_2}|2\rangle + e^{i\tau_2}|3\rangle]. \end{aligned} \tag{5.2}$$

Consequently, the effect of the operator $\hat{T}$ on this input is given as:

$$\begin{aligned}\hat{T}\psi_{23} &= \frac{1}{\sqrt{3}}\hat{T}[e^{i\tau_2}|2\rangle + e^{i\tau_2}|3\rangle] \\ &= \frac{1}{3}[e^{i\tau_2}(e^{i\tau_2}|1\rangle + e^{i\tau_1}|2\rangle + e^{i\tau_2}|3\rangle + e^{i\tau_2}(e^{i\tau_2}|1\rangle + e^{i\tau_2}|2\rangle + e^{i\tau_1}|3\rangle)].\end{aligned} \tag{5.3}$$

Without working out explicitly, the remaining relevant terms are given as:

$$\hat{T}\psi_{123} = \frac{1}{\sqrt{3}}[e^{i\tau_1}\hat{T}|1\rangle + e^{i\tau_2}\hat{T}|2\rangle + e^{i\tau_2}\hat{T}|3\rangle], \tag{5.4a}$$

$$\hat{T}\psi_{12} = \frac{1}{\sqrt{3}}[e^{i\tau_1}\hat{T}|1\rangle + e^{i\tau_2}\hat{T}|2\rangle, \tag{5.4b}$$

$$\hat{T}\psi_{13} = \frac{1}{\sqrt{3}}[e^{i\tau_1}\hat{T}|1\rangle + e^{i\tau_2}\hat{T}|3\rangle], \tag{5.4c}$$

$$\hat{T}\psi_{23} = \frac{1}{\sqrt{3}}\hat{T}[e^{i\tau_2}|2\rangle + e^{i\tau_2}\hat{T}|3\rangle], \tag{5.4d}$$

$$\hat{T}\psi_{1} = \frac{1}{\sqrt{3}}[e^{i\tau_1}\hat{T}|1\rangle], \tag{5.4e}$$

$$\hat{T}\psi_{2} = \frac{1}{\sqrt{3}}[e^{i\tau_2}\hat{T}|2\rangle], \tag{5.4f}$$

$$\hat{T}\psi_{3} = \frac{1}{\sqrt{3}}[e^{i\tau_2}\hat{T}|3\rangle], \tag{5.4g}$$

$$\hat{T}\psi_{0} = 0. \tag{5.4h}$$

Similar to (5.15), the Sorkin quantity vanishing is equivalent to the statement that

$$\hat{T}\psi_{123} - \hat{T}\psi_{12} - \hat{T}\psi_{13} - \hat{T}\psi_{23} + \hat{T}\psi_{1} + \hat{T}\psi_{2} + \hat{T}\psi_{3} - \hat{T}\psi_{0} = 0. \tag{5.5}$$

It is clear from our set of equations (5.3) and (5.4) that there are 2 positive contributions of $\hat{T}|j\rangle$, for j = 1,2,3, from unblocked and single blocked terms and 2 negative contributions from the double blocked terms. Thus, the Sorkin quantity does indeed vanish.

Although a clear experimental procedure to eliminate states has not yet been realised. A few potential concepts have been thought of.

Initially, an idea was to resonantly excite particular states to some other excited state that is not one of the four levels relevant to the tripod configuration. However, this would introduce alot of noise to the system in the form of spontaneous decays from this excited state.

Recoil-induced resonances [17] [18] could be used to couple target states to higher momentum states, either allowing them to escape a magnetic trap entirely or allows for distinguishing these faster states. A detailed plan utilising this has not been fully worked out yet.

5.2 Dephasing

Similar to the elimination of states, another possible realisation of a blocker could be by dephasing.

In the most general sense, suppose we have a superposition of states $|1\rangle$, $|2\rangle$, and $|3\rangle$ given by:

$$\psi = \alpha_1 e^{i\phi_1} |1\rangle + \alpha_2 e^{i\phi_2} |2\rangle + \alpha_3 e^{i\phi_3} |3\rangle , \tag{5.6}$$

where the coefficients and phases of the states can be any sensible value. Dephasing is proposed as a possible method to block states, corresponding to the operation:

$$D_k(|\psi\rangle \langle\psi|) = \frac{1}{2\pi} \int_0^{2\pi} |\psi\rangle \langle\psi| \, d\phi_k, \tag{5.7}$$

for the blocking of the slit labelled k. Note that this operation essentially averages the exponential function (with iϕ_k as exponent) over its period of 2π. Thus, it simply reduces all $e^{i\phi_k}$ terms to zero.

In matrix form, we have that,

$$|\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha_1|^2 & \alpha_1\alpha_2^* e^{i(\phi_1-\phi_2)} & \alpha_1\alpha_3^* e^{i(\phi_1-\phi_3)} \\ \alpha_2\alpha_1^* e^{i(\phi_2-\phi_1)} & |\alpha_2|^2 & \alpha_2\alpha_3^* e^{i(\phi_2-\phi_3)} \\ \alpha_3\alpha_1^* e^{i(\phi_3-\phi_1)} & \alpha_3\alpha_2^* e^{i(\phi_3-\phi_2)} & |\alpha_3|^2, \end{pmatrix} \tag{5.8}$$

which means blocking slits is just reducing the corresponding off-diagonal terms to zero.

To show that this operation still gives a vanishing Sorkin quantity, we first list all the relevant terms.

$$\hat{\rho}_0 = |\alpha_1|^2 |1\rangle\langle 1| + |\alpha_2|^2 |2\rangle\langle 2| + |\alpha_3|^2 |3\rangle\langle 3|, \tag{5.9a}$$
$$\hat{\rho}_1 = \hat{\rho}_0, \tag{5.9b}$$
$$\hat{\rho}_2 = \hat{\rho}_0, \tag{5.9c}$$
$$\hat{\rho}_3 = \hat{\rho}_0, \tag{5.9d}$$
$$\hat{\rho}_{12} = \hat{\rho}_0 + \alpha_1\alpha_2 * e^{i(\phi_1-\phi_2)} |1\rangle\langle 2| + \alpha_2\alpha_1 * e^{i(\phi_2-\phi_1)} |2\rangle\langle 1|, \tag{5.9e}$$
$$\hat{\rho}_{13} = \hat{\rho}_0 + \alpha_1\alpha_3 * e^{i(\phi_1-\phi_3)} |1\rangle\langle 2| + \alpha_3\alpha_1 * e^{i(\phi_3-\phi_1)} |3\rangle\langle 1|, \tag{5.9f}$$
$$\hat{\rho}_{23} = \hat{\rho}_0 + \alpha_2\alpha_3 * e^{i(\phi_2-\phi_3)} |2\rangle\langle 3| + \alpha_3\alpha_2 * e^{i(\phi_3-\phi_2)} |3\rangle\langle 2|, \tag{5.9g}$$
$$\hat{\rho}_{123} = \hat{\rho}_0 + \alpha_1\alpha_2 * e^{i(\phi_1-\phi_2)} |1\rangle\langle 2| + \alpha_1\alpha_3 * e^{i(\phi_1-\phi_3)} |1\rangle\langle 3| \tag{5.9h}$$
$$+ \alpha_2\alpha_3 * e^{i(\phi_2-\phi_3)} |2\rangle\langle 3| + h.c. \tag{5.9i}$$

Using these terms in (5.15), there are 4 $\hat{\rho}_0$ positive contributing terms, with another 4 negative contributions. Similarly, the off-diagonal terms exactly cancel out, so the Sorkin quantity is still zero.

An actual experimental procedure for dephasing has not been thought of yet. One possible idea is quantum decoherence by coupling the state to the environment.

5.3 Redistribution of States

Suppose now, that instead of removing states, we redistribute the population of the blocked states to the other unblocked states. We will eventually show that this scheme for blocking does not always work.

Redistribution could be achieved through spontaneous emission. Here, we propose to couple the states that are intended to be blocked by resonant lasers to the excited state in the tripod system. We can use polarised lasers such that transitions from the other states are forbidden by the selection rules for angular momentum. From this excited state, the electron will decay incoherently back to the ground states $|1\rangle$, $|2\rangle$, or $|3\rangle$ with transition probabilities related to the Clebsch-Gordan coefficients, r_1, r_2 and r_3 respectively. Therefore,

$$r_1 + r_2 + r_3 = 1. \tag{5.10}$$

An example of blocking state $|2\rangle$ is shown in Fig. **??**

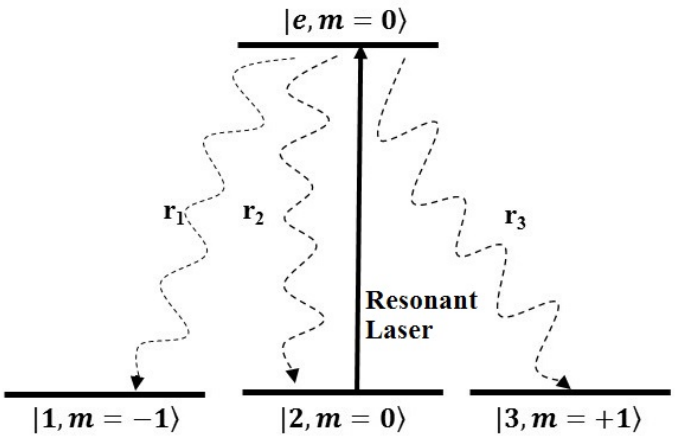

Figure 5.2: Blocking state $|2\rangle$ using the redistribution scheme. $|2\rangle$ is excited and then spontaneously decays back to any of the three ground states with rate r_1, r_2 and r_3.

5.3.1 1-Cycle

Suppose the resonant laser excites all populations previously in state $|k\rangle$. By "1-cycle", we mean that all the population of the excited state spontaneously decays to the ground states. The "1" refers to this process occurring only once. The result of the process is a mixed state, so we need to work with density operators. The full calculations are left to Appendix 7.1, and the final results are given here. After blocking state $|1\rangle$, the density operator for the system becomes:

$$\hat{\rho}_{23} = |c_2|^2 |2\rangle \langle 2| + |c_3|^2 |3\rangle \langle 3| + c_2 c_3^* |2\rangle \langle 3| + c_3 c_2^* |3\rangle \langle 2| + |c_1|^2 R, \tag{5.11}$$

where the term R can be thought of as the net effect of the spontaneous decay from the excited state, and is given by:

$$R = r_1 |1\rangle \langle 1| + r_2 |2\rangle \langle 2| + r_3 |3\rangle \langle 3| . \tag{5.12}$$

Similarly, blocking state $|1\rangle$ and $|2\rangle$ corresponds to:

$$\hat{\rho}_3 = |c_3|^2 |3\rangle \langle 3| + (|c_1|^2 + |c_2|^2) R. \tag{5.13}$$

Equations (5.11) and (5.13) agree with what one would expect from blocking states; the states that are not blocked remain unaffected but the blocked states (represented by the factor multiplying R) are redistributed based on R.

All the relevant density operators are listed below:

$$\hat{\rho}_1 = |c_1|^2 |1\rangle \langle 1| + (|c_2|^2 + |c_3|^2)R, \tag{5.14a}$$
$$\hat{\rho}_2 = |c_2|^2 |2\rangle \langle 2| + (|c_1|^2 + |c_3|^2)R, \tag{5.14b}$$
$$\hat{\rho}_3 = |c_3|^2 |3\rangle \langle 3| + (|c_1|^2 + |c_2|^2)R, \tag{5.14c}$$
$$\hat{\rho}_{12} = |c_1|^2 |1\rangle \langle 1| + |c_2|^2 |2\rangle \langle 2| + c_1 c_2^* |1\rangle \langle 2| + h.c. + |c_3|^2 R, \tag{5.14d}$$
$$\hat{\rho}_{23} = |c_2|^2 |2\rangle \langle 2| + |c_3|^2 |3\rangle \langle 3| + c_2 c_3^* |2\rangle \langle 3| + h.c. + |c_1|^2 R, \tag{5.14e}$$
$$\hat{\rho}_{13} = |c_1|^2 |1\rangle \langle 1| + |c_3|^2 |3\rangle \langle 3| + c_1 c_3^* |1\rangle \langle 3| + h.c. + |c_1|^2 R, \tag{5.14f}$$
$$\hat{\rho}_{123} = |c_1|^2 |1\rangle \langle 1| + |c_2|^2 |2\rangle \langle 2| + |c_3|^2 |3\rangle \langle 3|, \tag{5.14g}$$
$$+ c_1 c_2^* |1\rangle \langle 2| + c_2 c_3^* |2\rangle \langle 3| + c_1 c_3^* |1\rangle \langle 3| + h.c., \tag{5.14h}$$
$$\hat{\rho}_0 = R. \tag{5.14i}$$

To fit these densities into the Sorkin quantity, we need the probability for a general measurement $\hat{m}$, i.e. $P_{i,j,k} = \langle m| \rho_{i,j,k} |m\rangle$. However, if we were to carefully observe the third order Sorkin quantity defined in (3.4), we notice that there is no zeroth term. A possible justification for $\hat{\rho}_0$ is that unlike the triple slit experiment, blocking all 3 slits does not give a zero measurement based on this scheme.

From the relation between the density operators and the probability for a measurement, having the Sorkin quantity (3.4) to vanish is equivalent to the following sum to be zero:

$$\langle m| \hat{\rho}_{123} - \hat{\rho}_{12} - \hat{\rho}_{13} - \hat{\rho}_{23} + \hat{\rho}_1 + \hat{\rho}_2 + \hat{\rho}_3 - \hat{\rho}_0 |m\rangle . \tag{5.15}$$

It is clear from our equations above that this sum is indeed zero. Thus, we can actually use the 1-cycle scheme as our blocker, provided that such a short laser pulse can be realised. To describe the system on a long timescale, we generalise the idea to N-cycles and then to ∞-cycles.

5.3.2 N-Cycle

The N-cycle is a continuation from the 1-cycle, with the unblocked states again unaffected and the blocked states going through the excitation and decay cycle N times. Essentially, this is a recursion N times. The same subscript notation is used to indicate which states are unblocked and the leading number denotes the N-th cycle.

For the 2-cycle, blocking states $|2\rangle$ and $|3\rangle$ gives:

$$R_1^{(2)} = r_1 |1\rangle \langle 1| + (r_2 + r_3)(r_1 |1\rangle \langle 1| + r_2 |2\rangle \langle 2| + r_3 |3\rangle \langle 3|), \tag{5.16}$$

and blocking only state $|3\rangle$ gives:

$$R_{12}^{(2)} = r_1 |1\rangle \langle 1| + r_3 r_1 |1\rangle \langle 1| + r_2 |2\rangle \langle 2| + r_3 r_2 |2\rangle \langle 2| + r_3^2 |3\rangle \langle 3|). \tag{5.17}$$

While the 3-cycle gives:

$$\begin{aligned} R_1^{(3)} = {} & r_1 |1\rangle \langle 1| + (r_2 + r_3) r_1 |1\rangle \langle 1| + (r_2 + r_3)^2 r_1 |1\rangle \langle 1| \\ & + (r_2 + r_3)^2 (r_2 |2\rangle \langle 2| + r_3 |3\rangle \langle 3|), \end{aligned} \tag{5.18}$$

and,

$$\begin{aligned} R_{12}^{(3)} = {} & r_1 |1\rangle \langle 1| + r_1 r_3 |1\rangle \langle 1| + r_1 r_3^2 |1\rangle \langle 1| \\ & r_2 |2\rangle \langle 2| + r_2 r_3 |2\rangle \langle 2| + r_2 r_3^2 |2\rangle \langle 2| + r_3^3 |3\rangle \langle 3| . \end{aligned} \tag{5.19}$$

In general, for N cycles,

$$R_1^{(N)} = \sum_{i=1}^{N} (r_1 (r_2 + r_3)^{i-1} |1\rangle \langle 1|) + (r_2 + r_3)^{N-1} (r_2 |2\rangle \langle 2| + r_3 |3\rangle \langle 3|), \tag{5.20a}$$

$$R_{12}^{(N)} = \sum_{i=1}^{N} (r_3^{i-1} (r_1 |1\rangle \langle 1| + r_2 |2\rangle \langle 2|)) + r_3^N |3\rangle \langle 3| . \tag{5.20b}$$

5.3.3 ∞-Cycle

From the constraint (5.10) on the transition probabilites, we know that in the limit to infinity, $R_1^{(N)}$ converges because it is just a geometric progression.

$$\lim_{N\to\infty} R_1^{(N)} = B_1 = \frac{r_1}{1-r_2-r_3} |1\rangle\langle 1| \tag{5.21a}$$

$$= |1\rangle\langle 1|, \tag{5.21b}$$

$$\lim_{N\to\infty} R_{12}^{(N)} = B_{12} = \frac{r_1}{1-r_3} |1\rangle\langle 1| + \frac{r_2}{1-r_3} |2\rangle\langle 2| \tag{5.21c}$$

$$= \frac{r_1}{r_1+r_2} |1\rangle\langle 1| + \frac{r_2}{r_1+r_2} |2\rangle\langle 2|. \tag{5.21d}$$

In fact, we can easily see that we can extend this to all $R_{ijk}^{(N)}$ terms to obtain:

$$B_1 = |1\rangle\langle 1|, \tag{5.22a}$$

$$B_2 = |2\rangle\langle 2|, \tag{5.22b}$$

$$B_3 = |3\rangle\langle 3| \tag{5.22c}$$

$$B_{12} = \frac{r_1}{r_1+r_2} |1\rangle\langle 1| + \frac{r_2}{r_1+r_2} |2\rangle\langle 2|, \tag{5.22d}$$

$$B_{23} = \frac{r_2}{r_2+r_3} |2\rangle\langle 2| + \frac{r_3}{r_2+r_3} |3\rangle\langle 3|, \tag{5.22e}$$

$$B_{13} = \frac{r_1}{r_1+r_3} |1\rangle\langle 1| + \frac{r_3}{r_1+r_3} |3\rangle\langle 3|, \tag{5.22f}$$

$$B_{123} = 0, \tag{5.22g}$$

$$B_0 = r_1 |1\rangle\langle 1| + r_2 |2\rangle\langle 2| + r_3 |3\rangle\langle 3|, \tag{5.22h}$$

From the 1-cycle, we know that all the unblocked terms always cancels out in the calculation of the third order Sorkin quantity.

Notice, however, that arranging the relevant terms like in (5.15) does not yield a

zero. In fact, the Sorkin quantity for this case can be written explicitly as:

$$\begin{aligned} S_3^{(\infty)} = (|c_2|^2 + |c_3|^2 - r_1 - \frac{|c_1|^2 r_1}{r_1 + r_3} - \frac{|c_1|^2 r_1}{r_1 + r_2})|\left\langle m|1\right\rangle|^2 \\ + (|c_1|^2 + |c_3|^2 - r_2 - \frac{|c_2|^2 r_2}{r_1 + r_2} - \frac{|c_2|^2 r_2}{r_2 + r_3})|\left\langle m|2\right\rangle|^2 \\ + (|c_1|^2 + |c_2|^2 - r_3 - \frac{|c_3|^2 r_3}{r_1 + r_3} - \frac{|c_3|^2 r_3}{r_2 + r_3})|\left\langle m|3\right\rangle|^2, \end{aligned} \tag{5.23}$$

for a general measurement $|m\rangle\langle m|$. This quantity is, in general, non-zero except for specific r_1, r_2 and r_3 values.

Earlier, in the 1-cycle section, it was discussed that a zeroth term was introduced because blocking all 3 slits does not give a trivial measurement. This is true for blocking that eliminates states, but the "sum-over-histories" approach used in deriving the Sorkin terms is incompatible with redistribution of states. One way of seeing this is that while the sets of possible measurements from each unblocked slit is always fixed in the elimination of states, redistribution adds on to them depending on which slit is blocked. Then, the quantity in (5.23) is some residue from these extra contributions.

While the 1-cycle could potentially work as our blocker, the redistribution of states cannot directly verify the Born rule using the Sorkin quantity.

Chapter 6

Conclusions

The Born rule does not allow for three path or higher interference. An effective measure to quantify these interferences, termed the Sorkin quantity, is used. Based on previous works in the verification of the Born rule [3] [4] [5] [6] [9], there is still no significant evidence for possible violations of it. However, experimental bounds can be lowered and more observations can be recovered.

In this thesis, the theoretical basis of three path experiments has been discussed and the triple slit experiment is used as a reference point for future experiments. An atomic analog for the triple slit experiment, referred to as the atomic slit experiment, is proposed.

We have derived an implementation of an atomic tritter, using laser pulses of certain durations to generate an equal superposition of three states. An essential component to this tritter is the tripod configuration of electron energy levels. Atoms such as strontium or rubidium could be used [19] [20].

Several ideas for the blocker are brought up but have yet to be implemented experimentally. To maintain consistency with the triple slit experiment where blocked photons do not reach the detector, the blocker must also remove states from the system. It is shown that if instead, the states are redistributed, this blocker is not always compatible with the Sorkin quantity and should thus be avoided.

6.1 Future Works

The contributions from non-classical paths in the triple slit experiment are negligible under reasonable assumptions such as in the far field regime or negligible slit thickness [11] [13]. However, it is still not clear how these concepts directly translate from optics to the atomic picture. A full description of the atomic slit experiment can be analysed using the Feynman path integral formalism to show parallels or dissimilarities in this regard.

Furthermore, in the realisation of the blocker, a potential concept using the phenomena of recoil-induced resonances [17] [18] can be investigated. Target states can be coupled to higher momentum states, eventually allowing for them to either escape a magnetic trap entirely or be distinguished from the other states.

There is still work to be done in the theory for the atomic slit experiment. It is worth noting that three path experiments are not only restricted to the triple slit and atomic slit. Thus, developing a theory of the atomic slit experiment can lead to more interesting variations of three path experiments.

Chapter 7

Appendix

7.1 Density Operators fro redistribution

Working out the density operators associated with the blocking of states through spontaneous emission for the 1-cycle, we start with not blocking any state. This is then,

$$\begin{aligned}\hat{\rho}_{123} &= |\psi_{123}\rangle \langle\psi_{123}| \\ &= |c_1|^2 |1\rangle \langle 1| + |c_2|^2 |2\rangle \langle 2| + |c_3|^2 |3\rangle \langle 3| \\ &\quad + c_1 c_2^* |1\rangle \langle 2| + c_1 c_3^* |1\rangle \langle 3| + c_2 c_3^* |2\rangle \langle 3| + h.c.\end{aligned} \tag{7.1}$$

For the blocking of state 1, we first write down the renormalised superposition of states $|1\rangle$ and $\langle 2|$:

$$|\psi_{23}\rangle = \frac{c_2}{\sqrt{1-|c_1|^2}} |2\rangle + \frac{c_3}{\sqrt{1-|c_1|^2}} |3\rangle . \tag{7.2}$$

Then the density operator is given by:

$$\begin{aligned}\hat{\rho}_{23} &= (1-|c_1|^2)\,|\psi_{23}\rangle\langle\psi_{23}| + |c_1|^2 R \\ &= (1-|c_1|^2)(\frac{|c_2|^2}{1-|c_1|^2}\,|2\rangle\langle 2| + \frac{|c_3|^2}{1-|c_1|^2}\,|3\rangle\langle 3| + \frac{c_2 c_3^*}{1-|c_1|^2}\,|2\rangle\langle 3| + h.c.) \\ &\quad + |c_1|^2 R \\ &= |c_2|^2\,|2\rangle\langle 2| + |c_3|^2\,|3\rangle\langle 3| + |c_1|^2 R.\end{aligned} \tag{7.3}$$

For the blocking of states $|1\rangle$ and $|2\rangle$,

$$|\psi_3\rangle = \frac{c_3}{\sqrt{1-|c_1|^2-|c_2|^2}}\,|3\rangle\,, \tag{7.4}$$

so the density operator is,

$$\begin{aligned}\hat{\rho}_3 &= (1-|c_2|^2-|c_3|^2)\,|\psi_3\rangle\langle\psi_3| \\ &= (1-|c_2|^2-|c_3|^2)\frac{|c_3|^2}{1-|c_1|^2-|c_2|^2}\,|3\rangle\langle 3| + (|c_1|^2+|c_2|^2)R \\ &= |c_3|^2\,|3\rangle\langle 3| + (|c_1|^2+|c_2|^2)R.\end{aligned} \tag{7.5}$$

Blocking all three states corresponds to exciting all the population to the excited state and allowing it to spontaneously decay. Then, the density operator for blocking all states is,

$$\hat{\rho}_0 = R. \tag{7.6}$$

7.2 Electromagnetically Induced Transparency (EIT)

Under certain conditions in the Λ configuration, dark states may form. These are states that have become decoupled from the driving fields [21]. Consider again our interaction Hamiltionian for the Λ system. For two photon resonance, i.e.

$\delta = 0$,

$$\hat{H}_{int} = \frac{\hbar}{2} \begin{pmatrix} 0 & 0 & \Omega_1 \\ 0 & 0 & \Omega_2 \\ \Omega_1^* & \Omega_2^* & 2\Delta \end{pmatrix}, \tag{7.7}$$

and define θ and ϕ by:

$$\tan\theta = \frac{\Omega_1}{\Omega_2}, \tag{7.8a}$$

$$\tan 2\phi = \frac{\sqrt{\Omega_1^2 + \Omega_2^2}}{\Delta}. \tag{7.8b}$$

This system has eigenstates given by:

$$|d^+\rangle = \sin\theta \sin\phi \,|1\rangle + \cos\theta \sin\phi \,|2\rangle + \cos\phi \,|e\rangle\,, \tag{7.9a}$$

$$|d^0\rangle = \cos\theta \,|1\rangle - \sin\theta \,|2\rangle\,, \tag{7.9b}$$

$$|d^-\rangle = \sin\theta \cos\phi \,|1\rangle + \cos\theta \cos\phi \,|2\rangle - \sin\phi \,|e\rangle\,. \tag{7.9c}$$

The state $|d^0\rangle$ does not contain the excited state and thus is the dark state. If the atom is in this eigenstate, it cannot get excited to $|e\rangle$ and will remain in this state.

7.3 Tripod systems in atoms

While this thesis does not focus too much on the experimental details, possible options for tripod systems are strontium and rubidium [19] [20]. A diagram of the Rubidium tripod is shown in Fig. 7.1.

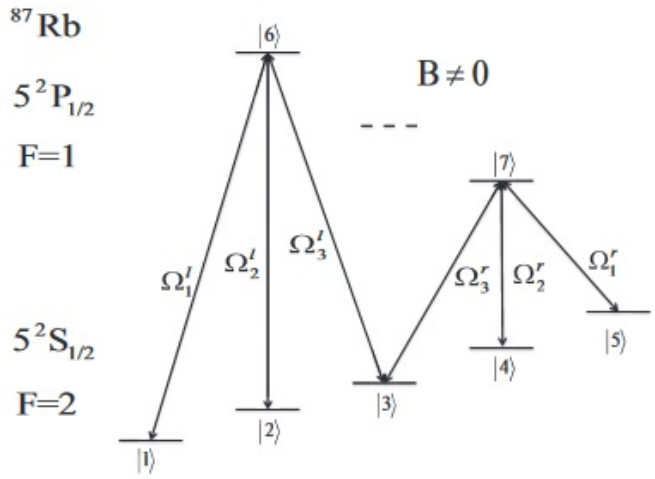

Figure 7.1: Two-tripod scheme for the D_1 line of Rubidium-87 [19]. An external magnetic field is applied and the states are shifted via the Zeeman effect.

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